Transportation Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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To cite this article:
Cinzia Cirillo, Renting Xu, Fabian Bastin (2016) A Dynamic Formulation for Car Ownership Modeling. Transportation Science 50(1):322-335. http://dx.doi.org/10.1287/trsc.2015.0597

TRANSPORTATION SCIENCE
Vol. 50, No. 1, February 2016, pp. 322–335
ISSN 0041-1655 (print) | ISSN 1526-5447 (online)
http://dx.doi.org/10.1287/trsc.2015.0597

A Dynamic Formulation for Car Ownership Modeling

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Discrete choice models are commonly used in transportation planning and modeling, but their theoretical basis and applications have been mainly developed in a static context. In this paper, we propose an estimation technique for analyzing the impact of technological changes on the dynamic of consumer demand. The proposed research presents a dynamic formulation that explicitly models market evolution and accounts for consumers' expectations of future product characteristics. The timing of consumers' decisions is formulated as a regenerative optimal stopping problem where the agent must decide on the optimal time of purchase. This model frame will be further improved by modeling the choice from a set of differentiated products whose characteristics randomly change over time. The framework proposed is developed and applied in the context of car ownership.

Keywords: dynamic decisions; discrete choice models; car ownership
History: Received: April 2012; revisions received: March 2013, September 2013, June 2014; accepted: September 2014. Published online in *Articles in Advance* May 15, 2015.

1. Introduction

The rising cost of energy, the heavy congestion on metropolitan motorways, and the pollution in urban areas caused by the use of private cars, enormously affects our economies and lifestyles. Increasing awareness about these problems is expected to dramatically affect mobility habits and to create opportunities for changes in the automotive industry over the next five to 10 years. More efficient and less pollutant vehicles will be available in the marketplace and new opportunities will be created for alternative energy sources. Estimation techniques for analyzing the impact of technological improvements and rapid changes in energy costs are necessary to understand the mobility of tomorrow and future preferences over vehicle characteristics. Static models based on disaggregate data are usually used to model and forecast car ownership in transportation planning; however, they are limited to providing a conditional in-sample analysis of counterfactual experiments.

In this paper we propose dynamic choice models to investigate demand for new vehicles (gasoline, electric, and hybrid) over the course of a finite temporal horizon. The model intends to capture not only choices relative to car types but also the optimal time of purchase. Dynamic models are also important to evaluate intertemporal substitution effects; in this framework consumers delay the adoption of a new car because of the expectation of better technologies or lower prices. The hypothesis that behavior is not affected by past and future states may cause the overestimation of market shares for new vehicle technologies and the failure of both public policies and private investments. In addition, the method aims at measuring the effects of innovations on consumers' preferences. An industry evolution model is proposed to account for changes in the products provided by the vehicle manufacturers or path dependence and dynamic in energy prices. In summary, our approach models forward-looking agents, industry evolution, heterogeneous products, and repeated purchases over time.

This paper begins with a discussion on modeling methodologies applied to the car market. Advantages and disadvantages of existing methods are outlined, and research cases and real studies are presented. A description then follows on the econometric techniques able to deal with dynamics in car ownership choices. A comprehensive modeling framework

is formulated; it includes the consumer utility specification, the definition of the dynamic programming problem, the industry evolution equation, and the optimization algorithm. The model is estimated and its performance tested on simulated data and real data collected from a stated preference survey conducted in Maryland. The innovative aspects of the model proposed together with the main contributions (with respect to earlier examples in economics) are outlined and their applications to other cases in transportation are anticipated.

2. Existing Car Ownership Applications

Car ownership and car users preferences are traditionally modeled with demand models, using one of two possible forms: aggregate or disaggregate. Aggregate car ownership models are mainly of three types: (1) time series models (Dargay and Gately 1999), (2) cohort models (Madre and Pirotte 1991), and (3) car market models (European Commission (DGII), Standard and Poors DRI and CES-KULeuven 1999). Aggregate models for car ownership allow dynamic specifications and have been used to estimate short- and long-run income elasticities. However, they usually include limited sociodemographic variables and do not explicitly model vehicle type. Aggregate models for car travel are important to understand the historic evolution of travel demand in response to changes in circumstances and economic factors. Panel surveys are then needed to trace the same individuals over time and to model the dynamics of choice behavior at the individual/household level. In this context, pseudo-panel models have been proposed to circumvent the need for panel data and their associated problems (Dargay and Vythoulkas 1999). One important limitation of pseudo-panel is that averaging over cohorts transforms the values of variables into cohort means, thereby losing information about the variation among households within each group.

Disaggregate car ownership and type choice models have been extensively developed and applied in the last three decades in several countries: the United Kingdom (Whelan 2001), the Netherlands (HCG 1993), Norway (TØI 1990), Australia (Hensher et al. 1992), the United States (Mannering and Winston 1985), etc. Their success is due to their behavioral foundations and to the possibility of including a large number of policy variables, as well as car types and use. Car ownership decisions can also be linked to a range of other travel choices allowing impact on car ownership of external factors (i.e., public transport cost and accessibility, congestion price, urban form, and density) to be represented. An integrated random utility maximization framework for personal-use cars and light trucks has been proposed and successfully implemented by Train (1986). The model system consists of several submodels that describe the number of vehicles owned, the class and vintage of each vehicle, and the miles traveled by each vehicle. The model was estimated on cross-sectional data extracted from the National Transportation Survey (NTS) augmented with data on the characteristics of makers and models. Using the inputs from the static model estimated on 1978 data, personal vehicle holdings, vehicle miles traveled, and fuel consumption were simulated over time and predictions were obtained for the years 1980–2000.

Mannering and Winston (1985) present a dynamic model similar to Train (1986) that also accounts for stationarity, state dependence, brand preference, and brand loyalty in car ownership, and utilization behavior. The model system is characterized by the presence of lagged dependent variables. The vehicle-type choice model and the utilization model contain in fact dummy variables indicating the use of the "same vehicles" or of the "same make" across up to two periods in time. Kitamura and Bunch (1990) estimated ordered-response probit models for panel data extracted from the Dutch Panel Survey and performed tests of heterogeneity versus true state dependence. The same data set has been used to study dynamics in car ownership choice, both at intertemporal dimensions (resistance to change in ownership levels due to uncertainty of financial position) and intratemporal dimensions (acquired taste for a certain lifestyle) by Nobile, Bhat, and Pas (1997). Mohammadian and Miller (2003) proposed a market-based decision-making process, and a transaction approach was applied to solve inconsistency in observed choices. Decision makers are assumed to face four possible choices each year: add a new vehicle to the fleet, dispose of one vehicle, trade one of the vehicles in the fleet, or do nothing. A mixed (random parameters) logit model was used to investigate the effects of heterogeneity in the dynamic transaction model and distinguish between heterogeneity-based and state-dependence-based explanations for the observed persistence in choice behavior.

Duration models for the time between vehicle transactions (and the type of transaction: disposal, replacement, acquisition, and scrappage) have also been used to explain the total number of cars in a household (Bhat and Pulugurta 1998). Gilbert (1992) studied how households adjust their car holdings when preferences or circumstances change because of adjustment costs. She proposed statistical methods known as duration or hazard models to estimate the distribution of automobile ownership lengths and to estimate the effects of characteristics of the car, household, and macroeconomic variables.

Rashidi, Mohammadian, and Koppelman (2011) used panel data (10 waves from 1989 to 2002) collected in Seattle and its surrounding areas to calibrate a model system of hazard-based equations in which timing of residential relocation, job relocation, and vehicle transaction are the endogenous variables. Duration models for vehicle transaction are very demanding in terms of data requirements, because panel or retrospective data are necessary for model estimation. Duration models are particularly attractive for short- to medium-term forecasts; for longer term, changes in the supply of car types can affect the validity of the predictions.

Dynamic models have been mainly used to predict if a household will make a vehicle transaction. Smith, Hensher, and Wrigley (1991) used a four waves panel data of household vehicle transaction to develop a dynamic discrete choice model assuming that outcome sequences through time are of the beta-logistic form. Transaction behavior is described as the binary choice between replacing versus keeping the car. Transactions involving new car types (i.e., alternative-fuel vehicles, efficient or smaller vehicles) need stated preference (SP) data to be modeled (Hensher and Greene 2001). Brownstone, Bunch, and Train (2000) used revealed preference (RP) and two waves of SP to estimate alternative fuel vehicles demand. SP data remained critical for obtaining information about attributes not available in the marketplace, but pure SP models gave implausible forecasts, hence the use of joint models. Abbe, Bierlaire, and Toledo (2007) have studied household vehicle usage to forecast future vehicle emissions, including the potential gains from alternative fuel vehicles. The study is based on an SP survey regarding electric vehicle usage collected on a mail-back survey in California. All existing studies based on SP data aim at forecasting market shares for new car types and individual preferences, but are incapable to predict when choices will happen over time. A dynamic structural approach that incorporates an optimal stopping problem has been applied by Schiraldi (2011) to study demand for automobiles. The model accounts for consumers' heterogeneity, expectations about future products, and endogeneity of prices. Replacement decisions of consumers in the presence of a secondhand market are considered and the distribution of transaction costs estimated. The model is applied in the Italian context to evaluate the impact of scrappage subsidies. However, the model is based on aggregate historical data, does not allow attributes to change dynamically over time, and is based on the assumption of infinite time horizon.

Recent research has proposed discrete continuous extreme value model formulations to analyze the choice of vehicle type/make/model and usage. Household demographics, household location characteristics, built environment attributes, household head characteristics, and vehicle attributes are used to explain vehicle holdings and use (Spissu et al. 2009). Bhat and Sen (2006) applied a multiple discrete-continuous extreme value (MDCEV) model to analyze holding and use of multiple vehicle types, exploiting data gathered from the 2000 San Francisco Bay Area travel survey. In Fang (2008), two models, a reduced-form Bayesian multivariate probit and Tobit (BMOPT) model and an MDCEV model derived from utility maximization, are applied to model households' vehicle holding and usage decisions in California. The system of BMOPT is composed of a multivariate ordered probit model and a multivariate Tobit model. The ordered probit is used to capture household decisions on a number of vehicles in each category. Within this framework, vehicles are categorized into fuel efficient (cars) and fuel inefficient vehicles (trucks), allowing the analysis of possible environmental and energy saving policy implications. Existing discrete-continuous models for car ownership and use are calibrated on cross-sectional data and do not account for dynamic effects.

It can be concluded that previous studies on dynamics for car ownership have focused on state dependency (Kitamura and Bunch 1990; Nobile, Bhat, and Pas 1997; Mohammadian and Miller 2003), new car type adoption using SP data (Hensher and Greene 2001; Brownstone, Bunch, and Train 2000; Abbe, Bierlaire, and Toledo 2007), and have mainly modeled influence on behavior of past experience (Mannering and Winston 1985). In the transportation literature, very little exists to model future expectations and optimal timing to make a vehicle transaction. Addressing this problem with discrete choice models requires multiperiod travel surveys, elaborate models, and estimation procedures. This study intends to overcome these difficulties and provides a general framework to be applied in the context where individual behavior is influenced by future states and where alternatives and their attributes are expected to change over time.

3. Summary of Dynamic Model in Economics

Dynamic discrete choice models were first developed in economics and related fields; they are generally applied to evaluate price and elasticities, intertemporal substitution, and the welfare gains from industry innovations. In dynamic discrete choice structural models, agents are forward looking and maximize expected intertemporal payoffs; the consumers get to know the rapidly evolving nature of product attributes within a given period of time and different products are assumed to be available on the

market. Changing prices and improving technologies have been the most visible factors in numerous new markets for durable goods. As a result, a consumer can either decide to buy the product or to postpone the purchase at each time period. This dynamic choice behavior has been treated in a series of different research studies; for a comprehensive survey on dynamic discrete choice structural models we refer to Aguirregabiria and Mira (2010). In this review of dynamic discrete choice models, we only focus on the aspects that are relevant for modeling car ownership preferences over time.

A general model for the analysis of discrete choices over time was formalized by Heckman in the early 1980s. The model, based on panel data, accommodates time-varying explanatory variables, serial correlation patterns for unobservable attributes, and interrelations among decisions taken at different times (Heckman 1981).

Rust (1987) first formalized the optimal stopping problem and estimated the time to replace a used bus engine. It is a single agent problem describing the purchase time decision, over a set of products with homogeneous attributes (bus engines with different models). Melnikov (2013) expanded this model to represent the decision of whether to buy a computer printer or to postpone the purchase based on the expected evolution of the product quality and price. In Melnikov's framework, the products are heterogeneous and consumers are homogeneous. Error terms are assumed to be independently distributed across consumers, products, and time periods; furthermore, the purchase is only made once in the consumers' lifetime. Lorincz (2005) extended Melnikov's optimal stopping problem with a persistent effect. Customers who already had a product may choose to upgrade it (i.e., upgrade the operating systems). Carranza (2010) examined the digital camera market and proposed a logit utility model with one time purchase and fully heterogeneous consumers. He estimated the joint distribution of consumers' preferences and parameters of the participation function, which was based on the observed number of purchases. The distribution of preference is defined as a continuous parametric distribution. Gowrisankaran and Rysman (2012) also analyzed the importance of dynamics when modeling consumer's preferences over digital camcorder products using a panel data set on prices, sales, and characteristics. This model combined the techniques developed by Berry, Levinsohn, and Pakes (1995) to model consumer heterogeneity in a discrete choice context and Rust Techniques for modeling optimal stopping decisions. This model explicitly accounted for dynamics in consumer behavior and allowed for unobserved product characteristics, repeated purchases, endogenous prices, and multiple differentiated products.

In transportation, dynamic applications of discrete choice models have mainly focused on individuals' previous actions (i.e., inertia) but do not consider future plans and random changes in external conditions, such as price and attributes of the targets under study. Therefore, the development of dynamic discrete choice models in transportation needs to be extended. Here we focus on possible applications of dynamic discrete choice models to car ownership for short- and medium-term planning (number of cars and type to own or purchase) and in particular on the potential of advanced technology (rapidly changing over time) on individual preferences and market evolution.

4. Car Ownership Formulation

4.1. General Consumer Stopping Problem

We consider a consumers set $\mathcal{I} = \{1, \ldots, M\}$ and time periods $t = 0, 1, \ldots$. In each time period t, consumer i has two options:

1. To buy one product $j \in \mathcal{J}_t$ and obtain a terminal period payoff u_{ijt}, where $\mathcal{J}_t = \{1, \ldots, J_t\}$ is the set of products available at time t.

2. To postpone and obtain a one-period payoff $c_{it}(x_{it}, q_{it}; \theta_i, \alpha_i)$ where x_{it} is an attributes vector for individual i at time t, e.g., sex, education, income, age, etc., q_{it} is the vector of characteristics of the owned product, θ_i, and α_i are parameter vectors for x_{it} and q_{it}, respectively.

Contrary to most of the cases existing in the literature, the decision process continues even when the consumer decides to buy as the proposed framework holds for repeated purchases. More precisely, we describe the car ownership problem with a regenerative optimal stopping problem: when the individual reaches the terminal state, the decision process is restarted, and some variables of the problem, such as current vehicle age and mileage, are reinitialized. Note here that the term regeneration can be taken in its usual statistical meaning (Ross 1997), so it is sufficient to focus on a sequence of choices from one regeneration time to the next one in the following discussion.

It is here assumed that the choice set $\mathcal{J}_t$ is consistent in each time period t, so we can drop the subscript t from $\mathcal{J}_t$ and J_t. Using bold font for random variables, and normal font for their realizations, we express the payoff $\mathbf{b}_{ijt}$ as a random utility function

$$\mathbf{b}_{ijt} = u(x_{it}, d_j, \mathbf{y}_{jt}, \theta_i, \gamma_i, \lambda_i, \boldsymbol{\epsilon}_{ijt}), \tag{1}$$

where

- $x_{it}, \theta_i \in \Re^Q$ are defined as above.
- $d_j \in \Re^K$ is a vector of static attributes for potential choice j and γ_i is a vector of parameters related to d_j.

- $\mathbf{y}_{jt} \in \Re^H$ is a random vector of dynamic attributes for product j at time t; e.g., energy (typically fuel) cost per mile,[1] purchase cost, environment incentives, etc., and λ_i is a vector of parameters related to $\mathbf{y}_{jt}$. Moreover, we summarize these attributes with a single random vector $\mathbf{y}_t = (\mathbf{y}_{1t}, \ldots, \mathbf{y}_{Jt})$.
- $\boldsymbol{\epsilon}_{ijt}$ is an individual-specific random term, whose components are independently and identically generalized extreme value (GEV) distributed among individuals and periods. We assume $\boldsymbol{\epsilon}_{ijt}$ to be independent of $\mathbf{y}_t$.

Although this formulation can be extended to a mixed GEV kernel (Bastin, Cirillo, and Toint 2006), the parameters here are assumed to be the same over individuals, i.e., $\theta_i = \theta$, $\alpha_i = \alpha$, $\gamma_i = \gamma$, and $\lambda_i = \lambda$, $i = 1, \ldots, M$.

We assume a two-step decision process, in which, at each time period, the consumer first decides to buy or postpone the purchase until the optimal time period τ, that is the time when the consumer decides to buy instead of postponing; then, the consumer chooses the product j_τ^* that maximizes utility (1) from $\mathcal{J}$. The consumer deciding to buy or postpone is the optimal stopping problem at time t

$$D_t(\mathbf{b}_{i1t}, \ldots, \mathbf{b}_{iJt}, c_{it}) = \max_\tau \left\{ \sum_{k=t}^{\tau-1} \beta^{k-t} c_{it} + \beta^{\tau-t} E_{\mathbf{y}_\tau}\left[\max_{j\in\mathcal{J}} \mathbf{b}_{ij\tau}, \,\middle|\, y_t \right] \right\}, \quad (2)$$

where

- β is a discount factor in $[0, 1)$;
- c_{it} is the payoff function of individual i's attributes and the characteristics of current product owned by i when choosing to postpone the purchase, as defined above.

It is important to note that the expectation in (2) is taken with respect to the industry evolution $\mathbf{y}_t$ and D_t remains a random function because of the terms $\boldsymbol{\epsilon}_{ijt}$ present in the random utility functions. Let $\mathbf{v}_{it} = \max_{j\in\mathcal{J}} \mathbf{b}_{ijt}$. We can directly rewrite (2) as

$$D_t(\mathbf{v}_{it}, c_{it}) = \max_\tau \left\{ \sum_{k=t}^{\tau-1} \beta^{k-t} c_{it} + \beta^{\tau-t} E_{\mathbf{y}_\tau}[\mathbf{v}_{i\tau} \mid y_t] \right\}.$$

According to the previously described assumption about $\boldsymbol{\epsilon}_{ijt}$, $\mathbf{v}_{it}$ is Gumbel distributed with a scale factor equal to 1 and r_{it} is the mode of distribution of $\mathbf{v}_t$. We also stress that if $\tau = t$, the right-hand term in (2) reduces to $\mathbf{v}_{it}$. It is then easy to see the consumer's decision can be transformed from (2) into

$$D_t(\mathbf{v}_{it}, c_{it}) = \max\{\mathbf{v}_{it}, c_{it} + \beta E_{\mathbf{y}_{t+1}}[D_{t+1}(\mathbf{v}_{i,t+1}, c_{i,t+1}) \mid y_t]\}. \quad (3)$$

[1] This allows us to summarize car consumption and current fuel price into one attribute.

Based on (3), we see that the decision process simply consists on buying at time t, or delay the purchase over one period, taking the payoff c_{it} plus the discounted future return. This is a standard regenerative optimal stopping problem, with a stopping set given by

$$\boldsymbol{\Gamma}(y_t) = \{\mathbf{v}_{it} \mid \mathbf{v}_{it} \geq \mathbf{W}_i(y_t)]\}, \quad (4)$$

where $\mathbf{W}_i(y_t)$, the reservation utility level for individual i, is defined as

$$\mathbf{W}_i(y_t) = c_{it} + \beta E_{\mathbf{y}_{t+1}}[D_{t+1}(\mathbf{v}_{i,t+1}, c_{i,t+1}) \mid y_t]. \quad (5)$$

Using (5), (3) can be simplified as

$$D_t(\mathbf{v}_{it}) = \max\{\mathbf{v}_{it}, \mathbf{W}_i(y_t)\}.$$

We assume the random terms $\boldsymbol{\epsilon}_{ijt}$ take specific realizations when selecting an individual i, meaning that the individual i has access to all values of his or her utility function, except the dynamic attributes of j. In other words, the vectors $\boldsymbol{\epsilon}_{ijt}$ are simply the unobserved factors, and as a consequence, the value of D_t is fixed for a given i

$$D_t(v_{it}) = \max\{\mathbf{v}_{it}, W_i(y_t)\}. \quad (6)$$

Therefore, the consumer i will buy some product at time t only when $v_{it} > W_i(y_t)$. If i is randomly drawn from the population, the analyst can compute the probability of postponing the purchase until the next period as

$$\begin{aligned}\pi_{i0t}(y_t) &\stackrel{\text{def}}{=} P_{it}[D_t(\mathbf{v}_{it}) = W_i(\mathbf{y}_t) \mid y_t] = P_{it}[\mathbf{v}_{it} \leq W_i(y_t)] \\ &= F_v(W_i(y_t), y_t) = e^{-e^{-(W_i(y_t)-r_{it})}}. \end{aligned} \quad (7)$$

Note the probability is taken with the set of random variables $\boldsymbol{\epsilon}_{ij\ell}$, for $\ell = t, t+1, \ldots$, i.e., the variables unobserved by the analyst, but with known values for individual i.

The probability of the product adoption is $h(y_t) = 1 - \pi_{0t}(y_t)$, and the product-specific purchase probability is

$$\begin{aligned}\pi_{ijt}(y_t) &\stackrel{\text{def}}{=} P_{it}[D_t(v_{it}) = \mathbf{v}_{it} \cap \mathbf{v}_{it} = u_{ijt} \mid y_t] \\ &= P[u_{ijt} \geq u_{ikt}, \forall k \neq j \cap u_{ijt} \geq W_i(y_{it})] \\ &= P[u_{ijt} \geq W_i(y_{it}) \mid u_{ijt} \geq u_{ikt}, \forall k \neq j] \\ &\quad \cdot P[u_{ijt} \geq u_{ikt}, k \neq j] \\ &= P[v_{it} \geq W_i(y_t)] P[u_{ijt} \geq u_{ikt}, k \neq j] \\ &= h(y_{jt}) P[u_{ijt} \geq u_{ikt}, k \neq j]. \end{aligned} \quad (8)$$

4.2. Industry Evolution

As expressed in §4.1, y_{jt} represents the evolution of the product j's attributes and the market environment. Because the future appears uncertain to the consumer, we represent this evolution by means of a stochastic process, a usual tool in economy and energy pricing (see, e.g., Glasserman 2004; Hervé 2011). More specifically, $\mathbf{y}_{jt}$ here is assumed to follow a normal diffusion process as in Melnikov (2013)

$$\mathbf{y}_{j,t+1} = \mu(y_{jt}) + L(y_{jt})\boldsymbol{\nu}_{j,t+1},$$

where

- $\boldsymbol{\nu}_{jt}$ $(j = 1, \ldots, J,\ t = 1, \ldots)$ are i.i.d. multivariate standard normal random vectors;
- $\mu(\mathbf{y}_{jt})$: $\Re^H \to \Re^H$ and $L(\mathbf{y}_{jt})$: $\Re^{H\times H} \to \Re^{H\times H}$ are continuous and have Jacobian matrices for almost every y_{jt}; $L(\mathbf{y}_{jt})L(\mathbf{y}_{jt})^T = \Sigma(\mathbf{y}_{jt})$, the variance-covariance matrix of the random vector $\mathbf{y}_{j,t+1}$; $\Sigma(y_{jt})$ is semidefinite positive, for almost every y_{jt};
- $\lim_{n\to\infty} \beta^n \mu^n(y_{jt}) < +\infty$, where $0 \le \beta < 1$, $\mu^0(y_{jt}) = \mu(y_{jt})$ and $\mu^n(y_{jt}) = \mu(\mu^{n-1}(y_{jt}))$.

The random vector $\mathbf{y}_{j,t+1}$ therefore follows a multivariate normal distribution of mean $\mu(y_{jt})$. It can for instance be specified as a random walk with drift η_j

$$\mathbf{y}_{j,t+1} = \psi_j y_{jt} + \eta_j + L\boldsymbol{\nu}_{j,t+1}.$$

For simplicity, it is assumed that ψ_j and η_j are the same over all of the alternatives (if for instance the industry evolution is the same over all alternatives). Therefore, $\mu(y_{jt}) = \psi y_{jt} + \eta$ and $L(y_{jt}) = L$. The model can however be easily extended to the case when the industry evolution is product dependent.

4.3. Dynamic Estimation Process

We first summarize the parameters to be estimated in the dynamic car ownership problem:

- θ, a vector of stationary consumer preference parameters related to individual attributes x_{it}.
- γ, a vector of parameters related to attributes for potential choice d_j.
- λ, a vector of parameters related to dynamic attributes of product j, y_{jt}. $\lambda = (\psi, \eta, L)$.
- β, the discount factor, set to 1 for simplicity.
- α, a vector of parameters for characteristics of current owned car q_{it}.

The parameters' estimation is performed by maximizing the likelihood function

$$\mathcal{L}(\theta, \gamma, \lambda, \beta, \alpha) = \prod_{i=1}^{M}\prod_{t=1}^{H} P_{it}[D_t(\mathbf{v}_{it}) \mid \theta, \gamma, \lambda, \beta, \alpha], \quad (9)$$

where the probabilities are taken with respect to the distributions of the variables $\boldsymbol{\epsilon}_{ijt}$, as in (7) and (8), given the values of the parameters, and H designs the number of time periods where we collect the observations.

Maximum log-likelihood estimation method is used to optimize the function (9). First π_{i0t} must be obtained from (7) and then π_{ijt} can be calculated. The key point during the whole process is to figure out how to calculate the expected utility. Here we consider the approximation of this infinite horizon problem by a finite horizon scenario tree. We argue that this technique, quite common in dynamic programming (see, e.g., Bertsekas 2005), but also stochastic programming (Shapiro, Dentcheva, and Ruszczyński 2009), has a better behavioral rooting because individuals can only project themselves in a limited term horizon, and the use of scenarios is a typical and well-founded approximation approach for multi-period expectations.

At each time period, the respondent is assumed to have a perspective about future scenarios in the short-term horizon, which is characterized by the alternatives' attributes changing over time. Therefore, when calculating the expectation utility, the analyst should account for the possible market conditions in the respondent's future scenarios. The time horizon is defined on a limited number of time periods. As a simple illustration, let us suppose that, starting from the generic time period t, the respondent faces two possible alternatives, buy a car of a certain type and not buy; at $t+1$, each of the two scenarios from time t generates another two buy and not buy scenarios, for a total of four scenarios. Thus, the decision process is formulated by means of a scenario tree (see Figure 1). This scenario tree constitutes the base for the calculation of the expected utility. An example is provided here on the procedure adopted to calculate $\pi_{i0,0}$ and $E_{\mathbf{y}_1}[D(\mathbf{v}_{i1}) \mid i]$, that we denote for simplification purposes by $E[D_1]$ since all of the expectations in the example are taken for individual i, and the time step is summarized in the index of D.

Assumption 1. *At each time period, the respondent has an expectation over a limited number of future time periods, which is limited to two in order to reduce the number of leaves in the tree scenarios. At time period* 0, *the respondent can anticipate the possible alternative characteristics for time periods* 1 *and* 2. *The expectation at time period* 3 *is set to* 0 ($E[D3] = 0$); *respondents are assumed to have no knowledge of time period* 3 *when they are called to decide at time period* 0. *Other expressions for the* $E[D_3]$ *could be used within the devised framework, but it would require more investigation to properly identify the most adapted. Nevertheless, we stress that this expectation should not involve future stages, in order to comply with the assumption that individuals look a fixed number of periods ahead.*

- Computation of $E[D_1]$. Since the reservation utility $W_i(y_0) = c_0 + E[D_1]$ can be calculated according to (5), $E[D_1]$ must be calculated first in order to get $\pi_{i0,0}$.

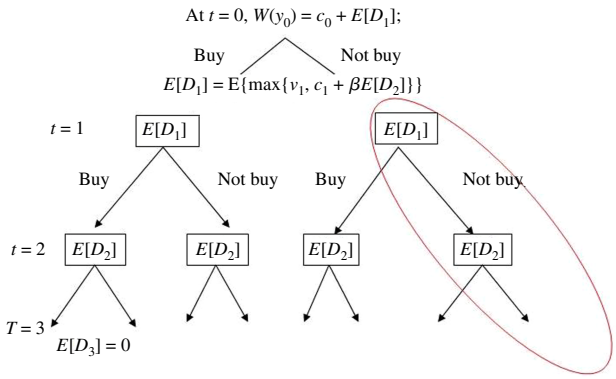


Figure 1 **(Color online) Scenario Tree**
Note. $E[D_t]$, t is the time period in this scenario tree.

At time 0, the respondent has two alternatives for successive time 1, buy the car with the highest utility or not buy (see Figure 1). The right side of the utility function $E[D_1] = E[\max\{v_1, c_1 + \beta E[D_2]\}]$ represents the utility of the not buy alternative; therefore, when calculating $E[D_2]$, we only take the terms corresponding to the right leaf of the tree in Figure 1. The calculation of $E[D_2] = E[\max\{v_2, c_2 + \beta E[D_3]\}]$ demands the same function to be calculated for period 3 ($E[D_3]$), which is assumed to be zero according to the above assumption.

The process of calculating $E[D_1]$ is recursive with known utility at the end of the perspective horizon (assumed to be two periods long in this formulation). Having calculated $E[D_1]$, reservation utility at time 0 $W_i(y_0)$ can be obtained.

• These steps can be repeated to calculate $\pi_{i0,1}$ with the assumption that the respondent can anticipate alternative characteristics for time periods 2 and 3 and supposing that $E[D_4]$ is equal to zero.

To summarize, for a receding horizon, a terminal value for the expected utilities has to be fixed, therefore the expectation of the last time period under the person's perspective, $E[D_T]$ must have a constant value. Since it is difficult to predict a particular value, we assume it to be zero. In the long term, the individual does not have enough information to predict the future; the individual cannot anticipate the utility of buying or postponing. Under this assumption, after a limited number of time periods, information on future market trends is just ignored.

5. Experiment Using Simulated Data

Synthetic households' choices over different time periods have been simulated to validate the proposed dynamic discrete choice formulation. The synthetic sample is composed of 200 individuals. Each of them is assumed to provide responses over 12 time periods starting from the current year; each time period is assumed to be six months long. A total of 2,400 observations are then generated. Respondents are assumed to choose between two alternatives: buy and not buy. If respondents choose to buy, they also need to decide which vehicle type they are going to buy among (1) gasoline vehicle, (2) hybrid vehicle, or (3) electric vehicle. Choice for each time period is calculated after comparing probabilities of the different alternatives generated. One important assumption made in the simulated process is that at each time period, previous decisions affect the alternatives in the current choice set. If the previous period choice is to buy, the current vehicle situation will be regenerated; the current vehicle will be the newly bought car and its age reset to zero. If the previous period choice is not buy, the current vehicle age is adjusted to reflect the fact that six months have passed since the last decision.

The variables in the simulated data set have been generated using the following criteria:

• Household characteristics. This set includes two discrete variables: number of family members and household income. The number of family members is assumed to be uniformly distributed in the range of 1–6. Household income varies on four levels of variation: (1) low, (2) medium, (3) medium high, and (4) high.

• Current vehicle characteristics. Car age is the only current vehicle attribute used in this experiment. Age for the first time period varies in the range of 0–10 years. Once age is generated randomly for the first time period, it is increased by 0.5 for each successive time period, unless a new car is bought, which will imply that the car age in the following time period is 0.5.

• Static potential vehicle attributes. Vehicle size is the attribute that characterizes new vehicles. Vehicle size is classified as (1) small, (2) medium, and (3) large.

• Dynamic attributes. We limit our analysis to one dynamic attribute, which is future gasoline price (expressed here in cents per gallon). The rationale behind the consideration of gasoline stems from the fact that consumers consider fuel price and energy-saving options when buying a car (Klier and Linn 2010), even if they do not rationalize the fuel budget on a given period (Turrentine and Kurani 2007). The function that defines gasoline price over time is generated using historical monthly prices observed in the past 30 years. Based on the assumption discussed in §4.3, the dynamic variable is assumed to follow a normal diffusion process and is specified as a random walk with drift. The calibrated function is

$$\mathbf{y}_{t+1} = 0.9757 y_t + 4.49 + \boldsymbol{\omega}_{jt},$$

where the drift η_j is set to 4.49, the auto-regressive factor ψ_j to 0.9757, and $\boldsymbol{\omega}_{jt}$ is normally distributed

Table 1 Model Estimation—Simulated Data

						MNL		Dynamic	
Alternative	Gas	Hybrid	Electric	Current	True values	Estim	t-stat	Estim	t-stat
ASC gasoline veh.	X				2.2	−0.63	0.6	2.79	6.6
ASC hybrid veh.		X			−0.4	−3.97	3.6	0.03	0.1
hh members		X	X		−0.65	−0.34	10.0	−1.06	32.3
hh income		X	X		0.35	−0.17	3.4	0.42	2.7
Veh. size (small)		X			1.9	2.15	12.4	2.63	26.2
Veh. size (large)			X		−0.1	−0.78	11.0	−0.13	2.9
Gas price	X	X			−1.4	−1.00	3.4	−2.12	19.2
Current veh. age				X	−1.8	−1.06	15.2	−1.63	60
N. obs.						2,400		2,400	
LL(0)						−3,327.10		−2,472.27	
LL(final)						−1,722.85		−685.05	
Likelihood ratio index						0.48		0.70	
RMSD						1.67		0.39	

Notes. ASC, Alternative specific constant; hh, household; RMSD, root mean square deviation.

with mean 0 and standard deviation equal to 16. Monte Carlo simulation with 1,000 draws is applied to generate gasoline fuel price for each time period. The average gasoline price for the initial time period is set to be \$3.5.

Using the simulated data and the specification defined above, two models were estimated: the static multinomial logit model and the dynamic model. The static model is estimated using the software Alogit (http://www.alogit.com/); the dynamic model algorithm is coded in C language and makes use of a number of optimization tools derived from AMLET (Another Mixed Logit Estimation Tool; http://amlet.slashbin.net/). In the static model, respondents are not considering future market evolution and possible vehicle technology improvements when making decisions at each time period. The model is simply formulated as a traditional multinomial logit (MNL) model with four alternatives; utilities include both static and dynamic variables, for consistency with the dynamic model formulation. From the results, we observe that the static model is unable to recover the true alternative specific constants and that the estimated coefficient for income is negative, which is inconsistent with the model assumptions. Other coefficients are recovered in sign but large biases remain especially with respect to the large vehicle size coefficient. The dynamic model clearly outperforms the static model. All coefficients have the correct sign and the alternative specific constant for gasoline vehicles is correctly estimated, whereas the alternative specific constant for hybrid vehicles is not significant. The gas price, which is the only dynamic variable estimated, is negative and highly significant. Results are summarized in Table 1. As expected, the fit of the model improves when considering the dynamic nature of the problem; the rho-squared increases from 0.48 to 0.7. Finally, in order to validate which model better recovers the true values, root mean square deviation (RMSD) is adopted as a measure of the differences between the true values and the predicted values. The bigger the RMSD, the poorer the model's ability to reproduce the true phenomenon. The RMSD is defined as

$$\mathrm{RMSD}(\hat{\theta}) = \sqrt{E((\hat{\theta}-\theta)^2)} = \sqrt{\frac{\sum_{i=1}^{n}(\hat{\theta}_i-\theta_i)^2}{n}},$$

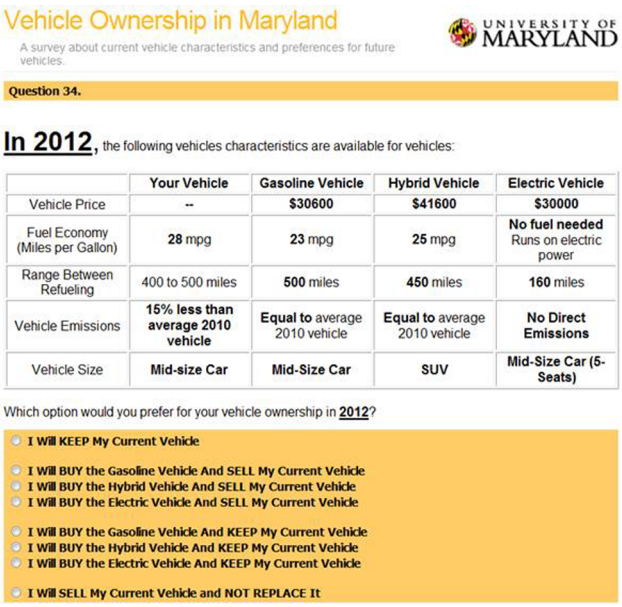
which n is the number of parameters. As can be seen from the last row in Table 1 the RMSD of the logit model is higher than the value obtained with the dynamic model.

6. Dynamic Model: Results from Real Data

6.1. Survey Design and Data Collection

Data collected in Maryland from an SP survey conducted in the fall of 2010 is used to estimate a dynamic model of new vehicle adoption. The original survey involved three stated choice experiments: vehicle technology, fuel type, and taxation policy; however, each respondent randomly received just one SP experiment. Each stated choice experiment generated multiple SP observations over a six-year time period, from 2010 to 2015. The variables in the scenarios changed from year to year when plausible. For example, vehicle price generally increased over time, the hybrid vehicle tax credit decreased with time, and the range for gasoline vehicles remained constant. Respondents were not informed about the characteristics of future vehicle options when deciding at a given time period, but were given the following instructions in order to collect more realistic behavior:

- Make realistic decisions. Act as if you were actually buying a vehicle in a real-life purchasing situation.
- Take into account the situations presented during the scenarios. If you would not normally consider



Vehicle Ownership in Maryland

A survey about current vehicle characteristics and preferences for future vehicles.

UNIVERSITY OF MARYLAND

Question 34.

In 2012, the following vehicles characteristics are available for vehicles:

	Your Vehicle	Gasoline Vehicle	Hybrid Vehicle	Electric Vehicle
Vehicle Price	--	$30600	$41600	$30000
Fuel Economy (Miles per Gallon)	28 mpg	23 mpg	25 mpg	No fuel needed Runs on electric power
Range Between Refueling	400 to 500 miles	500 miles	450 miles	160 miles
Vehicle Emissions	15% less than average 2010 vehicle	Equal to average 2010 vehicle	Equal to average 2010 vehicle	No Direct Emissions
Vehicle Size	Mid-size Car	Mid-Size Car	SUV	Mid-Size Car (5-Seats)

Which option would you prefer for your vehicle ownership in 2012?

- I Will KEEP My Current Vehicle
- I Will BUY the Gasoline Vehicle And SELL My Current Vehicle
- I Will BUY the Hybrid Vehicle And SELL My Current Vehicle
- I Will BUY the Electric Vehicle And SELL My Current Vehicle
- I Will BUY the Gasoline Vehicle And KEEP My Current Vehicle
- I Will BUY the Hybrid Vehicle And KEEP My Current Vehicle
- I Will BUY the Electric Vehicle And KEEP My Current Vehicle
- I Will SELL My Current Vehicle and NOT REPLACE It

Figure 2 (Color online) Vehicle Technology Experiment Example

buying a vehicle, then do not. But if the situation presented would make you reconsider in real life, then take them into account.

- Assume that you maintain your current living situation with moderate increases in income from year to year.

For more details on data collection and results obtained from the three SP experiments we refer to Maness and Cirillo (2012); an example of the SP scenarios presented to respondents is given in Figure 2.

For this modeling exercise, we use only the experiment devoted to vehicle technology, for which a higher number of observations are available. The vehicle technology experiment focused on presenting respondents with varying vehicle characteristics and pricing in order to discover preferences for vehicle technology. This experimental design consisted of four alternatives and five variables with a choice set size of eight. Four alternatives, current vehicle, new gasoline vehicle, hybrid electric vehicle (HEV), and battery electric vehicle (BEV), were shown to respondents. These vehicle platforms were chosen because they have a good chance for market share in the United States over the next five years. Gasoline vehicles are the traditional option, although hybrid electric vehicles have gained an important part of the car market share in the United States. Electric vehicles are not new to the marketplace, but recent advances in battery technology might point toward a cheaper electric car and to a wider commercialization. The variables of interest in the vehicle technology experiment included vehicle price, fuel economy, refueling range, emissions, and vehicle size. Vehicle price, presented in U.S. dollars, depended on the size of

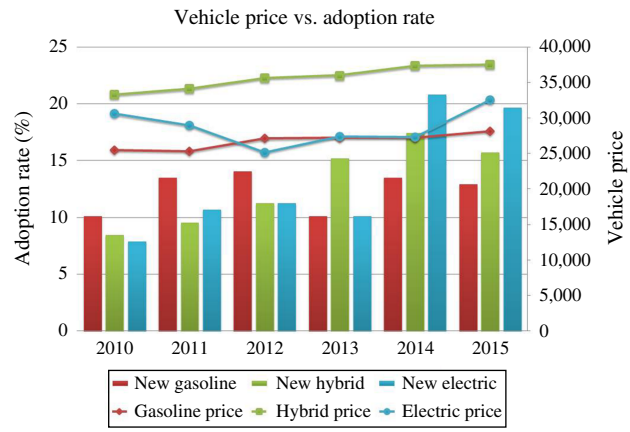


Figure 3 (Color online) Vehicle Price and Adoption Rate

the vehicle and increases annually. Fuel economy was presented in miles per gallon (MPG) for gasoline and hybrid vehicles. Refueling range was presented as the miles between refueling or recharging. Emissions were displayed as the percent difference in emissions in comparison to the average vehicle in 2010. Electric vehicles were stated to have no direct emissions. Vehicle sizes were based on the U.S. Environmental Protection Agency vehicle size system. The choice set for the vehicle technology experiment included all permutations of buying or not buying a new vehicle (gasoline, hybrid, or electric) and selling or retaining the current vehicle (Maness and Cirillo 2012).

Figure 3 shows market shares and car prices over time. New gasoline car price increases slightly in 2012 and then remains stable until 2015; the price of hybrid cars increases gradually, whereas the price of electric cars declines with respect to the initial value except for the year 2015. It appears that the interest in buying an electric car increases dramatically in 2014, when the market share for electric vehicles surpasses the shares of both new gasoline and hybrid vehicles. We would like to emphasize that these results are obtained from a small sample and that choices are based on artificial scenarios constructed by the analysts. As always in SP experiments, there is no guarantee that in real market conditions, consumers will make the same choices.

6.2. Static Model Results

The SP experiment on new technology vehicles has a structure that is similar to the one used to generate the simulated data. Four alternatives constitute the choice set: current car (not buy), new gasoline vehicle, hybrid vehicle, and electric vehicle. Variables included in the final model specification are gasoline car price, hybrid car price, electric car price, electric car range, miles per gallon, and current car age. Out of the 141 respondents that completed the survey, only 53 of them (those who declared to be willing to buy a new car)

were included in the final sample. It should be noted that although each respondent expressed preferences over 12 time periods spanning over six years, utilities and probabilities can be calculated for the first 10 time periods only. The remaining two alternatives are necessary to calculate the expectation over future time periods, consistent with the two steps look ahead modeling framework defined in the scenario tree of Figure 1.

A multinomial logit model was first estimated, using the specification in Table 2; for consistency, the static model is estimated on the same data set and using the same specification as for the dynamic formulation. All of the estimated coefficients have the correct sign and are statistically significant, except for the price of the nonelectric vehicles and the alternative specific constant (ASC) of the electric car. The ASC of both hybrid and electric cars are negative, whereas the ASC for keeping the current vehicle is positive. Vehicle age, specific to the current car alternative is negative, indicating that when the age of the present vehicle increases, the likelihood of keeping the vehicle decreases. The coefficient of the range variable, specific to the electric car alternative, is positive as expected. For fuel economy, respondents were split into groups based on their knowledge of the current vehicle fuel economy, measured in MPG. For respondents who knew their vehicle MPG, the difference between the current vehicle MPG and the MPG of the new vehicle was used for estimation. For respondents who did not know their vehicle MPG, the actual new vehicle MPG was used for estimation. Two coefficients for vehicle price were estimated; the dynamic price coefficient is specific to the alternative electric car, and the static price coefficient is common to both gasoline and hybrid vehicles. Electric car price is the only dynamic variable adopted in the dynamic formulation; this assumption was made to avoid possible confounding effects and to keep the model specification simple. From a behavioral standpoint, it is possible to say that more fluctuation and incertitude is expected for the electric car market; this would be reflected in the trends of electric vehicle prices. Our formulation is not consistent with the general rule that vehicle prices, as cost factors, should only have one coefficient estimated, so that the corresponding parameter can be used for the elasticity calculation, which will not be discussed in this context.

6.3. Dynamic Model Results

The dynamic model has the same specification of the MNL presented. The price of the electric car is treated as a dynamic variable and is assumed to vary according to a random drift. Electric car prices generated for the SP scenarios were used to calibrate the autoregression factor, drift, and variance of random draws under the hypothesis of residuals distributed as normal. After calibration, the dynamic variable function assumes the following form:

$$\mathbf{y}_{j,t+1} = -0.103 y_{jt} + 2.617 + \boldsymbol{\omega}_{jt}, \tag{10}$$

where y_{jt} corresponds to the electric car price (expressed in one ten-thousandth of a dollar), and $\boldsymbol{\omega}$ is a normally distributed term, with mean 0 and standard deviation 1.78. Respondent's perspective dynamic car prices in the scenario tree are then generated according to (10).

Unfortunately, the auto-regressive factor is very small; contrary to the one used for the simulated case and relative to the evolution of fuel price, which was estimated on a time series of real observations. That is due to the fact that scenarios in the survey were designed to be independent. Further research is needed to generate time dependent scenarios in the context of experimental design. The dynamic model estimation results are presented in Table 2 as well.

The estimated coefficients are all significant except the static vehicle price. It should be noted that the dynamic formulation estimates a significant coefficient for the alternative specific constant of the electric vehicle, which is expected to produce more realistic market share predictions (with respect to the static MNL model). The remaining coefficients keep the same sign obtained with MNL; changes in magnitude are registered although these effects are not dramatic. The fit of the model improves when considering the dynamic nature of the problem; the rho-squared increases from 0.22, the value obtained with the logit model to 0.42 for the dynamic model.

Coefficient estimates are used in application to calculate the prediction power of the models; the error norm calculated from the dynamic model (2.93) is smaller than the value obtained by applying the static model (3.24). Figures 4–7 present the observed and predicted market trends of gasoline vehicle, hybrid vehicle, electric vehicle, and keeping the current vehicle along the 10 time periods in the five years considered. The probability of keeping the current car is relatively high, starting around 70% in the first time period; acceptance of new vehicles starts already in early stages of the time horizon, although volatility is observed at some points. New gasoline vehicles, hybrid vehicles, and electric vehicles occupy smaller market shares (around 10% each) at the end of the five year periods; all new typologies become more popular after the fifth time period. Static models have a tendency to average choice probabilities over time and are incapable of recovering peaks in the demand function. More specifically, MNL underestimates the market share of the not buy alternative and dramatically overestimates the share occupied by electric

Table 2 Model Estimation—Stated Preference Data on Vehicle Technology

					MNL		Dynamic	
Alternative	Gas	Hybrid	Electric	Current	Estim	*t*-stat	Estim	*t*-stat
ASC hybrid veh.		X			−0.40	1.6	−1.09	4.05
ASC electric veh.			X		−0.50	0.9	1.18	1.94
ASC current veh.				X	1.52	3.2	−1.10	6.96
mpg known	X	X			0.052	4.0	0.078	6.20
mpg unknown	X	X			0.016	2.1	0.042	3.66
Veh. age				X	−0.097	4.3	−0.133	4.26
Veh. price (static)	X	X			−0.26	1.8	−0.062	0.46
Veh. price (dynamic)			X		−0.37	2.4	−1.01	5.37
Range			X		0.44	2.1	0.723	4.32
N. obs.					530		530	
LL(0)					−734.74		−1,683.09	
LL(final)					−614.66		981.43	
Likelihood ratio index					0.22		0.42	

Notes. ASC, Alternative specific constant; mpg, miles per gallon.

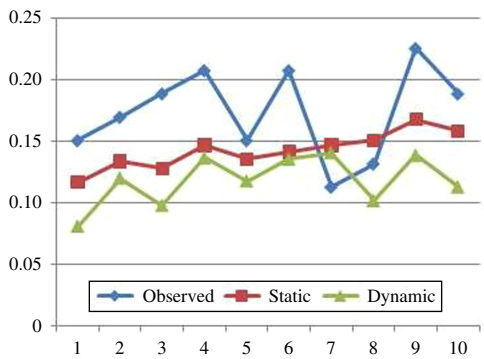


Figure 4 (Color online) Market Trend for Gasoline Car

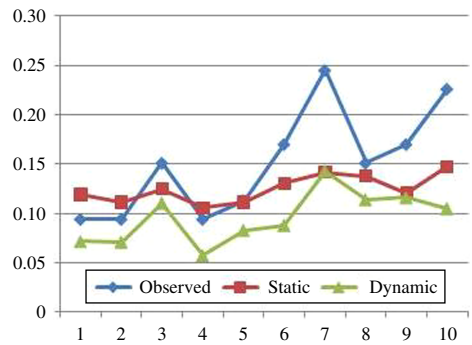


Figure 5 (Color online) Market Trend for Hybrid Car

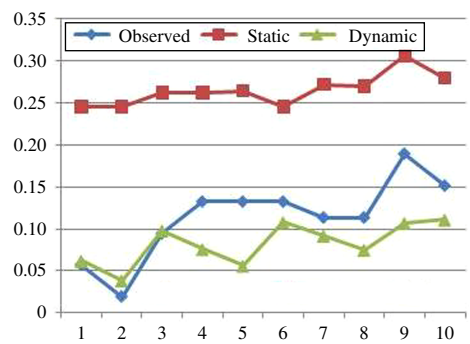


Figure 6 (Color online) Market Trend for Electric Car

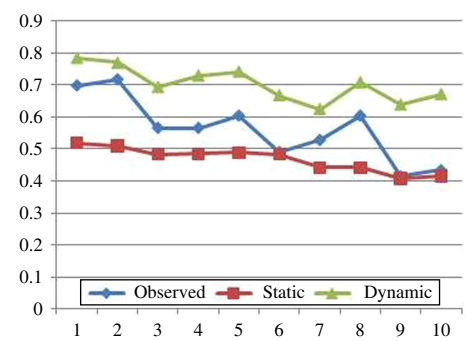


Figure 7 (Color online) Market Trend for Current Car

vehicles in the next five years; it predicts quite well the market for new gasoline vehicles and for hybrid vehicles. Dynamic model formulation overestimates the number of respondents keeping their current vehicles, but it is capable of reproducing the descending trend for this alternative. Dynamic discrete choice model (DDCM) does an excellent job in recovering market trends for the electric vehicles, which starts at 5% and terminates at around 10% in 2015. For the remaining alternatives, the DDCM captures the general behavior, but shows gaps in model prediction higher than those delivered by the static model.

7. Contributions

The main contributions of the research presented in this paper can be summarized as follows:

1. Static discrete choice models typically assume that individual utilities are linear functions of the alternatives and individuals' characteristics; attributes are not time dependent and can be defined for the actual situation (RP data) or future scenarios (SP data). Dynamic models estimated in this paper are nonlinear in utilities; utilities include information on both current alternatives and individual expectations about future alternatives.

2. The model proposed is more complex than traditional dynamic discrete choice models used in economics. Usually stopping problems are characterized by just two state options, assume homogeneous population, and choice sets are composed of just one product. More importantly, the consumer is considered out of the market when his or her status changes. In the problem modeled here, decision makers have more than one starting condition: each household can own zero, one, or multiple vehicles. Moreover, the population is heterogeneous, and every time a household decides to change his or her status, there are multiple alternatives available, each characterized by different vehicle technologies. A regenerative process allows for the consideration of multiple purchases.

3. The optimization problem for parameters' estimation is solved using a maximum-likelihood estimation method. Dynamic discrete choice models based on dynamic programming are usually estimated by using the nested fixed point optimization algorithm, which is valid under the restrictive assumption that the time horizon is infinite and that the operators are contractant (Rust 1987).

4. The dynamic discrete decision process is solved by generating a scenario tree from the underlying stochastic process. In our formulation, the stochastic problem is related to the nature of the attributes that change stochastically over scenarios and to the uncertainty in the individuals' future expectations; the optimization problem concerns individuals' utility maximization according to the random utility maximization paradigm. The optimization problem is expressed and solved in a recursive manner. Such model formulation and presentation, as well as the associated visual structure, has suggested a computational method of solution. This provides a graphically intuitive model construction and evaluation capabilities for transportation modelers who may be less familiar with stochastic modeling and algorithms.

5. A pilot survey has been designed and executed in order to estimate dynamic models in a real context and to test their performance with respect to traditional static models. Hypothetical scenarios are separated by six-month intervals and span a six-year period. Between scenarios, the vehicle and fuel attributes dynamically change to mimic marketplace conditions. Empirical analysis shows that respondents are able to create trade-offs between different vehicle technologies as well as the price of various fueling options. In addition, about 65% of respondents intend to buy a vehicle in the next six years; this result definitely shows that a potential market exists for new and more efficient gasoline cars and for electric and hybrid vehicles.

8. Conclusions

Increasingly, energy efficient and environmentally friendly highway transportation technologies are under development and could be available on the market in the near future. Estimation techniques for analyzing the impact of technological improvements and rapid changes in energy costs are necessary to understand the mobility of tomorrow and adapt the products of our car industry. This paper has developed a dynamic econometric model that accounts for the evolving characteristics of the products offered by the automobile industry and consumers' expectations of future vehicle quality. The timing of consumers' purchases is formalized as an optimal stopping problem where the agent (consumer) must decide on the optimal time of purchase. The modeling framework is further enriched by explicitly considering the consumer's choice from a set of different types of vehicles whose quality changes stochastically over time. The proposed approach extends the theory of discrete choice models on a temporal basis and improves existing dynamic discrete choice models based on a pure dynamic programming perspective. The modeling framework has been applied to both simulated and real data. In both cases, results show that dynamic models are superior to static models based on MNL; in particular, they are able to recover peaks in the demand evolution over time, whereas static models fail to detect dramatic changes because of the rapid mutation of external conditions.

We hope that this work will generate innovations in demand modeling and that it will be extended to other problems that are dynamic in nature. The following points indicate possible avenues for future research:

• The model formulation allows for just one dynamic attribute in the utility specification; the random walk is actually estimated on a univariate time series. The analysis should be extended to multivariate random walks.

• The number of scenarios considered for the calculation of the expected utility is limited to two. This is a rather restrictive assumption, made solely because our purpose here was to test the newly developed methodology. It would also be desirable to extend the time horizon over which the respondents consider future information about new alternatives.

• Data collection techniques should be improved in order to capture interdependency among successive observations in time. Methods to incorporate random walks into orthogonal experimental design (for SP data) should be developed.

• The dynamic model should be estimated on a revealed preference panel data set. TNS Sofres organizes and maintains a panel data set that follows the evolution of car ownership and car use since

1983. When working with RP data, one could expect a potentially larger bias because of neglecting price endogeneity. Future research is needed to assess if approaches similar to the one proposed by Train and Winston (2007) in the context of car ownership modeling can also be used in a dynamic context.

• From the optimization perspective, it would be interesting to compare the results obtained from maximum likelihood estimation with those obtained from the nested fixed point procedure and to demonstrate if the underlying hypotheses are valid for our case, which is developed for a finite horizon problem.

• This dynamic framework can be adapted and transferred to other case studies: dynamic pricing for revenue management, route choice behavior under dynamic tolling, and activity scheduling for activity-based analysis, just to cite a few.

Acknowledgments

This research has been supported in part by the National Science Foundation [CMMI-1131535]. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. The third author would like to thank NSERC for partial funding support.

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