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Dynamic discrete choice model for railway ticket cancellation and exchange decisions

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ABSTRACT

The increasing use of internet as a major ticket distribution channel has resulted in passengers becoming more strategic to fare policy. This potentially induces passengers to book the ticket well in advance in order to obtain a lower fare ticket, and later adjust their ticket when they are sure about trip scheduling. This is especially true in flexible refund markets where ticket cancellation and exchange behavior has been recognized as having major impacts on revenues. In this paper, we propose an inter-temporal choice model of ticket cancellation and exchange for railway passengers where customers are assumed to be forward looking agents. A dynamic discrete choice model (DDCM) is applied to predict the timing in which ticket exchange or cancellation occurs in response to fare and trip schedule uncertainty. The problem is formulated as an optimal stopping problem, and a two steps look-ahead policy is adopted to approximate the dynamic programming problem. The approach is applied to real ticket reservation data for intercity railway trips. Estimations results indicate that the DDCM provides more intuitive results when compared to multinomial logit (MNL) models. In addition, validation results show that DDCM has better prediction capability than MNL. The approach developed here in the context of exchange and refund policies for railway revenue management can be extended and applied to other industries that operate under flexible refund policies.

1. Introduction

Ticket cancellation and exchange behavior has significant impact on the revenue management (RM) system (Iliescu, 2008). In flexible refund markets, passengers are inclined to book their tickets in advance in order to obtain lower fares, and to exchange/cancel the tickets when changes in their schedule intervene. Moreover, the use of internet as a major ticket distribution channel has affected the behavior of customers who have now better access to fare information, and are becoming more strategic in their choices. Reliable predictions in cancellation and exchange decisions are believed to enable analysts to derive more efficient overbooking and refund/exchange policies. RM applications to air transportation have demonstrated to significantly reduce the number of empty seats on flights for which there is actually a potential demand (Neuling et al., 2004). Existing literature on choice modeling for revenue management (RM) have mostly ignored temporal effects in individual decision making. Although static models enable analysts to address the dependence of demand on the set of products offered by the provider, they are unable to model forward looking agents, who would typically wait and see before making the final decision.

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In this paper, we propose a dynamic framework based on discrete choice models developed in the context of railway revenue management. Dynamic discrete choice models have been firstly developed in economics and applied to study a variety of problems that include fertility and child mortality (Wolpin, 1984), occupational choice (Miller, 1984), patent renewal (Pakes, 1986), and machine replacement (Rust, 1987). In dynamic discrete choice structural models, agents are forward looking and maximize expected inter-temporal payoffs; the consumers get to know the rapidly evolving nature of product attributes within a given period of time and different products are supposed to be available on the market. The timing of consumers' purchases is usually formalized as an optimal stopping problem where the agent (consumer) must decide on the optimal time of purchase (Rust, 1987). To the authors' knowledge, this is the first attempt to incorporate dynamics in individual choices to revenue management modeling and in particular to formalize tickets' exchange and cancellation decisions for railway intercity trips. The railway operator in consideration offers fully refundable fare and provides flexibility in ticket exchange which makes ticket cancellation and exchange decision to be very crucial to the RM system. Passengers are incentivized to purchase ticket early and adjust their ticket later when they are more certain about trip schedules. The model proposed accounts for passengers' trip adjustment choice and explicitly specifies the probability of exchanging ticket as a function of the set of available exchange tickets. The choice set is constituted by all departure times offered by the railway operator between a specific origin destination pair.

The remainder of the paper is organized as follows. Studies on strategic models for customer purchasing behavior and specifically for ticket cancellation and exchange are reported in Section 2. In Section 3, we formulate a dynamic discrete choice model and we formalize the algorithm used for the dynamic programming problem under study. Data used for model estimation is presented in Section 4, together with descriptive statistics concerning ticket cancellation and exchange behavior. In Section 5 we offer some empirical evidence from the application of the proposed model. Finally, conclusions drawn from the empirical analysis and future research directions are outlined in Section 6.

2. Literature review

There is an emerging research effort toward dynamic frameworks that account for inter-temporal variability in choice modeling. Existing research on inter-temporal price variation that considers forward-looking consumers includes Stokey (1979), Landsberger and Meilijson (1985), and Besanko and Winston (1990). These papers are based on the assumptions that customers are present in the market throughout the entire season, and that the seller's inventory is practically unlimited. Customers purchase at most one unit during the season, and they optimally select the timing of their purchases so as to maximize individual surplus. Su (2007) studied a model of strategic customer by identifying four customer classes, different from each other in two dimensions: high versus low valuations and strategic (i.e., patient) versus myopic (impatient) behavior. The price path is assumed to be predefined by the seller, and after the specific pricing policy is announced, strategic consumers can weigh the benefits of waiting for a discount (if any is offered). The paper demonstrates that the joint heterogeneity in valuations and in the degree of patience is crucial in explaining the structure of optimal pricing policies.

Behavior of ticket cancellation and exchange is clearly influenced by demand uncertainty over time. Stokey (1979) showed that offering a single price can be optimal when inter-temporal differentiation is feasible, but assumes that consumers have perfect information on the future evolutions of their valuations. In Png's (1989), consumers face both uncertainty in their valuations as well as uncertainty about the capacity. Gale and Holmes (1992) examined advance purchase discounts where a monopoly firm offers two flights at different times and where consumers are assumed to not know their preferred flight in advance. In this study, advance purchase discounts are used to smooth the demand of the consumers with a low cost of time. Gallego and Phillips (2004) used a similar approach in their work on flexible products. Dana (1998) showed that advance purchase discounts may improve the revenues of price-taking firms when consumer demand is uncertain. In this case, firms in competitive markets can improve profits by offering advance purchase discounts. Shugan and Xie (2000) developed an inter-temporal consumer choice model for advance purchase which distinguishes the act of purchasing and consumption. The model accounts for buyer's valuation of services that depends on buyer states at the time of consumption and assumes the product capacity to be unlimited. In a later paper, Xie and Shugan (2001) extended this analysis of advance selling to the finite-capacity case and introduced a refund option. Ringbom and Shy (2004) proposed a model where consumers have the same deterministic valuation (maximum willingness to pay) for a certain service of product but different probabilities of showing up; capacity is assumed to be infinite and prices are endogenously given; results show that by adjusting partial refunds it is possible to endogenize the participation rates. Aviv and Pazgal (2008) considered an optimal pricing problem of a fashion-like seasonal good in the presence of strategic customers (forward-looking characteristics) with a time-varying valuation pattern. Customers have partial information about the availability of the inventory and their arrival is assumed to be time dependent. The system is characterized by a leader follower game under Nash equilibrium where customers select the timing of their purchase so as to maximize individual surplus while the seller maximizes expected revenue. Gallego and Sahin (2010) developed a model of customer purchase decision with evolution of trip schedule valuations over time. This analysis considers partial refundable fare based on a call option approach; each customer updates his/her valuation over time and decides when to issue and when to exercise options in a multi-period temporal horizon. Very recently, Baucells et al. (2016) have studied the wait-or-buy problem from a behavioral perspective. The core idea of their method is that the wait-or-buy decision reflects a multidimensional trade-off between the delay in getting an item, the likelihood of getting it, and the magnitude of the price discount. Although their formulation is different from ours, the wait-or-buy problem has similarities with the exchange-cancellation problem proposed in this paper in the context of revenue management.

Meanwhile, a number of studies on demand uncertainty have focused on the supply chain management approach. To our knowledge, in operations management literature, Spinler et al. (2002, 2003) are among the first who incorporated consumer's

uncertainty in valuations into revenue management, and the first to study partially refundable fares. Other studies on uncertain valuations for traditional revenue management problems include Levin et al. (2009), Yu et al. (2015), and Koenigsberg et al. (2008). There is also an emerging literature that deals with strategic consumers who develop expectations on future prices and product availability based on the observed history of prices and availabilities (Besanko and Winston, 1990; Liu and van Ryzin, 2008; Aviv and Pazgal, 2008). Latinopoulos et al. (2017) look at different theoretical perspectives of intertemporal choices and decision-making under uncertain prices. In particular, they model the response of EV drivers to dynamic pricing of parking-and-charging tariffs and explore the behavioral process that takes place in such a reservation system.

In the context of ticket cancellation and exchange models, a number of papers have been published in the past decade. Garrow and Koppelman (2004a) proposed an airline cancellation and exchange behavior model based on disaggregate passenger data; airline travelers no-show and standby behavior is modeled using a multinomial logit (MNL) model estimated on domestic US itineraries data. The approach enables the identification of rescheduling behavior based on passenger and itinerary characteristics and supports a broad range of managerial decisions. Variables used to identify passenger rescheduling behavior are traveler characteristics, familiarity to the air transportation system, availability of viable transportation alternatives, and trip characteristics. Garrow and Koppelman (2004b) extended their work by introducing a nested logit structure and demonstrated the benefit of directional itinerary information. The nested logit (NL) tree groups show, early standby, and late standby alternatives in one nest and no show alternative in another nest. The analysis emphasized the superiority of nested logit model structure over multinomial logit model and the importance of distinguishing between outbound and inbound itineraries. Iliescu et al. (2008) further expanded the work of Garrow and Koppelman (2004a, 2004b) by proposing a discrete time proportional odds (DTPO) model to predict the occurrence of ticket cancellation and exchange based on the Airline Reporting Corporation (ARC) data. The cancellation probability is defined as a conditional probability that a purchased ticket will be canceled in a specific time period given it survived up to that point (hazard probability). Results show that the intensity of cancellation is strongly influenced by the time from the ticket purchase and the time before flight departure as well as by other covariates (departure day of week, market, group size, etc.). Specifically, higher cancellation is observed for recently purchased ticket and ticket with close departure dates. Graham et al. (2010) adopted discrete time proportional odds (DTPO) model to investigate when and why travelers make changes to their airline itineraries. Analysis is based on a nine-month period panel data of university employees in Atlanta, US. The analysis focused on tickets issued less than 60 days before the outbound departure date. The use of panel data enabled the analysts to study how cancellation behavior differs by frequency of travel as well as by carrier. The deriving empirical analysis identifies the reasons why business travelers exchange their ticket, and concluded that differences exists between outbound and inbound itineraries, between exchange and cancellation rates for frequent and infrequent business travelers, across air carriers and timing when refund and exchange events occur. The results also indicate that the timings of cancellation exhibit a strong pattern, i.e., ticket changes are two to three time more likely to happen within the first week after purchase and are more likely to occur as the departure date approaches.

In summary, while many attempts have been made to understand the impact of choice behavior in revenue management, the issue of passenger uncertainty over trip scheduling has not been extensively explored. Behavior of ticket cancellation and exchange is clearly influenced by the evolution of passenger certainty about trip making over time. Specifically, none of the existing studies allows for departure time specific exchange decision in the cancellation and exchange model while accounting for inter-temporal behavior of passengers. Thus, our study aims to fulfill this gap.

3. Problem formulation

3.1. Passenger stopping problem

We consider a passenger set $\mathfrak{T} = \{1,...,M\}$ where each passenger $i \in \mathfrak{T}$ can be in one of the two possible states $s_{it} = \{0,1\}$ in time period $t \in \{0,1,...,T\}$. Passenger is considered to be in the decision process when $s_{it} = 0$ and out of the decision process when $s_{it} = 1$. In each time period t, passenger i in state $s_{it} = 0$ has two options:

1. To perform a ticket modification (either exchange or cancel). Once decided to adjust the ticket, the passenger makes the choice of $j \in \mathfrak{I}_t$ which is composed of exchange (departure time specific exchange decision at time period t) and cancel alternatives and obtain a terminal period payoff u_{ijt} (or final utility). The utility of exchange is primarily a function of fare difference between the original and the new ticket at time period t. The utility of ticket cancellation is primarily a function of trip characteristics, and the refund amount.
2. To keep the original ticket, denoted here with k, and obtain a one-period payoff U_{ikt} (or transitional utility); which is normalized to have a mean of zero before departure day and equal to c on departure day.

The two-step decision process assumes that, at each time period, the passenger decides whether to keep or change the ticket. The optimal time period in which passenger decides to change the ticket is denoted by τ, where the passenger chooses the ticket change alternative j_τ^* from $\mathfrak{I}_\tau$ that maximizes the utility. The passenger decision at time t is the optimal stopping problem:

$$D(u_{i1t},...,u_{iJt},i,k,t) = \max_{\tau} \left\{ \sum_{s=t}^{\tau-1} U_{ist} + E\left[\max_{j\in\mathfrak{I}} u_{ij\tau}\right] \right\} \tag{1}$$

Let $v_{it} = \max_{j\in Im} u_{ijt}$. The passenger will change the ticket if he/she does not expect a better utility in the next stages, otherwise

he/she will keep the ticket. Mathematically speaking, (1) is equivalent to define:

$$D(\upsilon_{it},U_{ikt}) = \max\{\upsilon_{it},U_{ikt} + E[D(\upsilon_{i,t+1},U_{ik,t+1})]\} \tag{2}$$

where, by convention, $E[D(\upsilon_{i,T+1},U_{ik,T+1})] = 0$ as the passenger cannot travel after the time horizon T. Let define the reservation utility as:

$$W_{it} = U_{ikt} + E[D(\upsilon_{i,t+1},U_{ik,t+1})] \tag{3}$$

and consider the optimal policy:

$$D(\upsilon_{it},U_{ikt}) = \begin{cases} \upsilon_{it} & \text{if } \upsilon_{it} \geqslant W_{it} \\ W_{it} & \text{otherwise} \end{cases} \tag{4}$$

With these notations, the problem can be simplified as:

$$D(\upsilon_{it},U_{ikt}) = \max(\upsilon_{it},W_{it}). \tag{5}$$

Our model can be viewed as a generalization of the discrete time proportional odds model (DTPO) introduced by Iliescu et al. (2008), as both discretize the horizon in time steps, and at each step, the user performs a binary decision to keep the ticket or not, conditionally to choices to keep it at the previous steps. We however allow the decision process to incorporate uncertain factors, and the individual bases this decision on the future expected utility. In other terms, the choice is influenced by the decision maker's expectation of the future. Moreover, one the decision to cancel the ticket made by the individual, he/she has the opportunity to choose another departure time, and consequently buy a new ticket. This possibility directly affects the individual choice, but is not considered in the DTPO model.

3.1.1. Keep ticket probability

The passenger i will keep the ticket at time t when $W_{it} \geqslant \upsilon_{it}$. Let π_{i0t} denotes the probability of keeping ticket until the next period, which can be written as:

$$\pi_{i0t} = P[\upsilon_{it} \leqslant W_{it}] = P[\text{keep}|s_{it} = 0] \tag{6}$$

3.1.2. Change ticket probability

The probability of ticket change is $P[\text{change}|s_{it} = 0] = 1-\pi_{i0t}$ and the choice specific ticket change probability is:

$$\begin{aligned} \pi_{ijt} &= P[u_{ijt} \geqslant u_{ilt}, \forall\, l \neq j, \upsilon_{it} \geqslant W_{it}] \\ &= P[u_{ijt} \geqslant W_{it}|u_{ijt} \geqslant u_{ilt}, \forall\, l \neq j]P[u_{ijt} \geqslant u_{ilt}, l \neq j] \\ &= (1-\pi_{i0t})P[u_{ijt} \geqslant u_{ilt}, l \neq j] \end{aligned} \tag{7}$$

3.2. Objective function and parameters to estimate

The parameter estimation is performed by maximizing the likelihood function:

$$\mathscr{L}(\beta) = \prod_{i=1}^{M}\prod_{t=0}^{T} P_{it}[\text{decision}] \tag{8}$$

The decision probability is presented as:

$$P_{it}[\text{decision}] = P_{it}[\text{decision}, s_{it} = 0] + P_{it}[\text{decision}, s_{it} = 1] = P_{it}[\text{decision}|s_{it} = 0]P[s_{it} = 0] + P_{it}[\text{decision}|s_{it} = 1]P[s_{it} = 1] \tag{9}$$

The state s_{it} is observed in the data set, if the passenger has not changed the ticket, $P[s_{it} = 0] = 1$ and $P[s_{it} = 1] = 0$. Once the passenger changes the ticket, the passenger is considered to be out of the decision process, therefore $P[s_{it} = 0] = 0$ and $P[s_{it} = 1] = 1$. As a result, the complete likelihood function in this problem is:

$$\mathscr{L}(\beta) = \prod_{(i,t)\in V} P_{it}[\text{decision}, s_{it} = 0] \tag{10}$$

where $V = \{(i,t)|i \in \{1,...,M\}, t \in \{1,...,T\}$ and $s_{it} = 0\}$. The decisions include keeping the ticket and ticket change specific choice. Thus $P_{it}[\text{decision}, s_{it} = 0] = \{\pi_{i0t}, \pi_{ijt}\}$.

3.3. Dynamic estimation process

The estimation process is done with maximum likelihood estimation method. First π_{i0t} must be obtained in order to calculate π_{ijt}. The probability π_{i0t}, depends on W_{it} which can be calculated from: $W_{it} = U_{ikt} + E[D(\upsilon_{i,t+1},U_{ik,t+1})]$. W_{it} is composed of two parts: the utility of the current ticket attributes (U_{ikt}) and the expected utility in the next time period ($E[D(\upsilon_{i,t+1},U_{ik,t+1})]$). At each time period, the passenger is assumed to have a perception about the future scenarios, which are characterized by the alternative attributes changing over time. The expectation utility accounts for the possible market conditions in the passenger's perceived scenario; in our

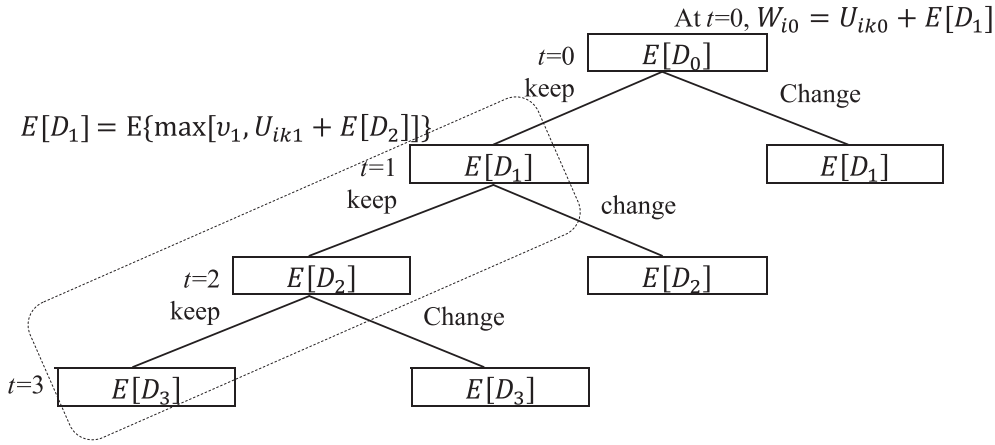

Fig. 1. Scenario tree.

specification, the fare of each departure time specific exchange decision has been selected as independent variable in the utility specification. Passenger is assumed to have a perception of future attributes on a limited number of time periods, denoted by T. At time period t, the passenger faces two alternatives, keeping the ticket or changing the ticket. The passenger will continue the decision process into the period $t + 1$ only if he had decided to keep the ticket in time period t. The decision process can therefore be characterized by a scenario tree with a unique pattern (shown in Fig. 1). This scenario tree constitutes the base for the expected utility calculation. The following steps describe the procedure to calculate $\pi_{i0,0}$ and $E[D(\upsilon_{i1},U_{ik1})]$ which will be indicated by $E[D_1]$ because all the expectations in the example are for individual i. The procedure for calculating the expected utility will be described in detail as follows:

- First, we assume that a passenger defines his/her expectation over a limited number of future time periods, which is limited to two in order to reduce the number of leaves in the scenario tree. At time period $t = 0$, the passenger can anticipate the future ticket characteristics (i.e. fare) from time period $t = 1$ and $t = 2$. The terminal time period expected utility $E[D_3] = 0$ because the passenger knows nothing for time period 3 when being at time period 0.
- Calculate $E[D_1]$. In order to obtain $\pi_{i0,0}$ from Eq. (6), the reservation utility (W_{i0}) is required. The reservation utility (W_{i0}) can be obtained from Eq. (3) $W_{i0} = U_{ik0} + E[D_1]$ which requires the calculation of $E[D_1]$. At time 0, the passenger has two alternatives for successive time 1, keep the ticket or change the ticket. The second term at the right hand side of the function $E[D_1] = \mathrm{E}\{\max[\upsilon_1,U_{ik1} + E[D_2]]\}$ represents the utility of keeping alternative; therefore when calculating $E[D_2]$, it is necessary that the term corresponded to the left leave of the tree be obtained (indicated by dash line in Fig. 1). The calculation $E[D_2] = \mathrm{E}\{\max[\upsilon_2,U_{ik2} + E[D_3]]\}$ demands the same function to be calculated for time period 3 ($E[D_3]$) which is assumed to be zero according to the above assumption. The process of calculating $E[D_1]$ is recursive with known utility at the end of the perspective horizon (assumed to two periods in this formulation). After $E[D_1]$ is calculated, reservation utility at time 0 (W_{i0}) can be obtained.
- This calculation procedure can be repeated to calculate $\pi_{i0,1}$ with the assumption that respondent can anticipate characteristics for time period 3 and $E[D_4] = 0$.

The reason that a terminal value for the expected utility has to be fixed at zero is because it is difficult to predict a particular value for the individual's perspective when future time period is far beyond his knowledge of information. This means that in the long term, the individual has not enough information to predict the future; passengers cannot anticipate the utility of keeping or cancelling the ticket. With this approach, after a limited number of time periods, information on future ticket fare attribute is just ignored.

4. Data analysis

The data set used for the analysis has been extracted from intercity railway ticket reservation records registered in March 2009. This data set contains 155,175 individual transactions expressed in terms of ticket purchase, cancellation, and exchange over time prior to departure. Ticket exchange decision is defined as the exchange of the original ticket for a new one and the payment of an additional cost depending on the operator's exchange policy. In our case study, passengers are not charged with exchange fee, but have to pay the difference between the new and the old ticket fare. In the case of ticket exchange, passenger either obtains a new ticket right away or after several time periods (repurchase). Ticket cancellation is defined as the final cancellation of the ticket with the passenger obtaining ticket refund depending on the operator's refund policy.

Table 1 shows the descriptive statistics derived from the dataset in use. Ticket exchange and cancellation account for 18.22% and 29.75% of the sample respectively. Single exchange and no more than two exchanges account for 80.82% and 95.79% of the exchange ticket respectively (14.73% and 17.46% of the sample). We observe that only 2.26% of the sample make an exchange prior

Table 1
Data overview.

Ticket exchange	No. reservation	% of exchange	% of total
1. Total exchange	28,280	100.00%	18.22%
1.1 Number of exchange			
Exchange (one time)	22,857	80.82%	14.73%
Exchange (one or two times)	27,088	95.79%	17.46%
Exchange (more than 2 times)	1193	4.22%	0.77%
1.2 Type of exchange			
Change Origin Destination (OD) (a)	4773	16.88%	3.08%
No change (either OD or departure)	7001	24.76%	4.51%
Reschedule departure day (b)	1406	4.97%	0.91%
Reschedule departure time	13,565	47.97%	8.74%
Reschedule departure day and time (c)	1539	5.44%	0.99%
Ticket Cancellation	No. reservation	% of cancel	% of total
2. Total final cancellation	46,158	100.00%	29.75%
2.1 Final cancellation after exchanged	3506	7.60%	2.26%
Total (Northbound, March 2009, Coach Class)	155,175		100.00%
Effective Sample (Total - (a) - (b) - (c))	147,457		95.03%

to ticket cancellation; thus in our model, we assume that passenger make ticket adjustment no more than once (either exchange or cancel). Based on this assumption, data are constructed to model the first exchange decision in case of multiple exchanges, and model final cancellation in case passenger both exchange and cancel. We do not consider passengers who change origin/destination or reschedule departure day because the share of these population is relatively low accounting for 3.08% and 1.90% (0.91% + 0.99%) of the sample respectively. Consideration of changes in origin/destination and departure day decisions requires the definition of a choice set that is significantly different across passengers and no information is available to construct a realistic choice set for each passenger. This results in the focused sample population to be composed of entire sample (155,175) subtracted by passengers with origin/destination change and departure day change (a, b, and c in Table 1) which results in 147,457 individual ticket reservation records of the sample.

The problem is further simplified by considering only passengers who made weekday trips from south end terminal station to 3 major destinations (named STA1, STA2, and STA3), and purchased the ticket 15 days before departure which results into a time horizon of 16 days for each decision maker (from 15 days before departure until departure day). This results in 696 valid individual passenger records for model estimation.

Based on the trip schedule revealed by these 696 passenger records, ticket fare of the original departure time and other departure times within the same departure day are constructed for each day over the decision horizon based on historical data. In each time period, if the passenger decides to change or cancel the current ticket, then the same passenger will no longer be in the decision process in the next time period; all observations occurred in the period after ticket exchange are excluded from the dataset. This results into a total of 7268 observed decisions valid for model estimation.

5. Experiment with real ticket reservation data

5.1. Model specification

The model specification considers 16 discrete time periods defined by $t \in \{0,1,...,15\}$ where t also represents the number of day from original ticket purchase. The first time period is the day when original ticket is purchased ($t = 0$), (day 1). The last time period is departure day ($t = 15$). At a given decision period, the future does not contain uncertain elements, so the expectation operator in (1) can be omitted and the model can be computed analytically. If some random process were to affect the future utilities and bring uncertainty after the current decision stage, we could approximate the expectations by Monte Carlo as discussed in Cirillo et al. (2016). The utility specification is defined as follows:

Exchange alternatives (5:00 AM to 7:00 PM)

$$\left.\begin{array}{l} U_{i5t} = \beta_{cost}(f_{5t} - f_{b0}) + \beta_{dfi.exc}t + \beta_{day16}day\ 16 + \beta_{ear.exc}early + \varepsilon_i \\ \cdots\cdots\cdots\cdots \\ U_{ijt} = \beta_{cost}(f_{jt} - f_{b0}) + \beta_{dfi.exc}t + \beta_{day16}day\ 16 + \beta_{ear.exc}early + \varepsilon_i \\ \cdots\cdots\cdots\cdots \\ U_{i19t} = \beta_{cost}(f_{19t} - f_{b0}) + \beta_{dfi.exc}t + \varepsilon_i \end{array}\right\}$$

$$U_{ict} = ASC_{cnl} + \beta_{gp}gp + \beta_{mr}mr + \beta_{ev}ev + \beta_{Mon}Mon + \beta_{Fri}Fri + \beta_{STA1}STA1 + \beta_{STA3}STA3 + \beta_{ref}f_{b0} + \beta_{dfi.cnl}t + \beta_{day1}day1 + \varepsilon_i$$

$$U_{ikt} = \begin{cases} c + \varepsilon_i & \text{if } t = 15 \\ \varepsilon_i & \text{if } t < 15 \end{cases} \tag{11}$$

For ticket exchange decision, the index j indicateso 15 exchange departure time (5:00 AM to 7:00 PM). The utility of exchange

Table 2
Estimation result: real data.

	Exchange	Cancel	Keep	MNL		Dynamic (2-SL)	
				Est	T-stat	Est	T-stat
ASC cancel		x		−6.297	12.9*	−3.652	57.1*
> 1 psg		x		−0.869	2.1*	−1.090	1.5
Orig Dept. time 5–9 am		x		0.143	0.8	0.639	1.2
Orig Dept. time 3–7 pm		x		−0.327	1.9	−0.760	1.4
Depart Monday		x		0.556	1.8	2.740	3.0*
Depart Friday		x		−0.286	1.8	−0.451	1.0
STA1 destination		x		−0.435	2.3*	−0.306	0.6
STA3 destination		x		0.557	2.5*	1.648	2.6*
Exchange cost	x			−0.011	19.3*	−0.026	3.7*
Refund		x		0.014	6.0*	0.042	9.6*
Keep (day 16)			x	1.885	11.0*	−3.547	12.8*
Day from issue	x			−1.217	35.3*	0.189	5.8*
Day from issue		x		0.163	5.9*	0.266	35.4*
Cancel (day 1)		x		5.629	18.3*	3.169	42.7*
Exchange (day 16)	x			17.050	30.5	1.578	10.2*
Early exchange	x			−3.299	24.5*	−1.751	12.1*
Log-likelihood (0)					−20,592		−4324
Log-likelihood (final)					−7629		−3117
Likelihood ratio index					0.63		0.28
No. individual					696		
No. observations					7268		

* Statistically significant at 5% significance level.

(U_{ijt}) includes exchange cost which is defined as the difference between the original fare (f_{b0}) and new fare (f_{jt}) at time t, and day from issue (dfi) which is the number of day from original ticket purchase equal to t where $t = 0$ on the day of original purchase and $t = 15$ on departure day. The utility of cancel (U_{ict}) includes alternative specific constant (ASC), dummy of group traveler (gp), dummies of original departure in the morning (5:00–9:00 AM) (mr), and evening (3:00–7:00 PM) (eV), dummies of original departure on Monday and Friday, dummies of STA1 and STA3 destination, refund (f_{b0}), the number of days passed since the ticket was issued (dfi), and dummy of cancel on day 1 ($day\ 1$). The utility of keep (U_{ikt}) is defined in two cases. In the last time period ($t = 15$) passenger deciding to keep the ticket obtain an utility that includes the constant term relative to the utility of traveling with the original ticket. In other time periods ($t < 15$) the systematic term of the keep utility is normalized to zero. ε_{ijt} is the random error term for each alternative at a given time period. ε_i is the individual error term which is assumed to be constant across all observations produced by the same respondent.

5.2. Estimation result

The results obtained from model estimation are shown in Table 2. Most of the variables are statistically significant at 5% confidence level. The results obtained from the dynamic model show negative sign in a number of variables associated with cancel decision which are: group traveler (party size includes more than one passenger), evening departure (original departure time from 3:00 to 7:00 PM.), original departure on Friday, and STA1 destination. This indicates low tendency of passenger with these characteristics to cancel their ticket. On the other hand, passengers with morning departure (original departure time from 5:00 to 9:00 AM.), original departure on Monday, and STA3 destination have a positive sign for the corresponding structural coefficients, indicating that passengers with these characteristics have higher likelihood to cancel the ticket. In particular, passengers traveling early in the week and traveling alone (typically associated with business travelers) are more likely to cancel their ticket which is in line with the results of Iliescu (2008).

The exchange cost and refund have the expected sign indicating disutility associated with paying additional cost to exchange ticket and the utility of receiving refund when ticket is canceled respectively. The variable of keeping the ticket on departure day (day 16) shows negative sign which could be explained by the fact that the fare of the original ticket possessed by the passenger is higher compared to a ticket hypothetically exchanged to other departure times. Another reason could be that passengers intentionally want to exchange/cancel the ticket but could not find an alternative departure time which economically matches their schedule.

The day from issues (number of days since the original ticket is purchased) has positive sign for the variable associated with exchange and cancel decision; this indicates that it is preferable for passengers to adjust their ticket later. This is line with expectations and consistent with results obtained by Iliescu (2008), who found that the odds of ticket change increase as the departure date approaches due to a strong effect of “last minute” change of plan. More specifically, the day from issue coefficient for the cancel decision has larger magnitude compared to the day from issue coefficient for the exchange decisions. This is intuitive based on this operator’s refund policy; passengers are fully refunded if the ticket is exchanged up to one hour before departure, while late tickets exchange is possible but limited by the uncertainty about seats availability.

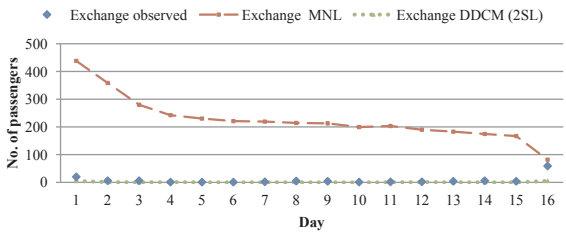

Fig. 2. Validation of exchange decision: real data.

The dummy variables of cancel on the original purchase date (day 1) and exchange on the departure day (day 16) show large magnitude indicating that a high number of cancellation and exchange occurs on the day they purchase ticket and on the departure day respectively. These results are in line with Iliescu (2008) and Graham et al. (2010) which found that ticket changes are more likely to happen in recently purchased ticket (especially within the first week) and are more likely to occur as the departure date approaches. Finally, the variable associated with early exchange (exchanging to departure time earlier than original ticket) shows negative sign which indicates that passengers gain less utility when making early exchange compared to later exchange (which is the base case).

5.3. Model validation

To test the prediction capabilities of the model proposed, the resulting coefficients of the model have been used to replicate the choice observed in the sample. Figs. 2–4 briefly summarize the predictions over different time periods (days) where exchange decisions are aggregated for all exchange departure times. The validation results show that the DDCM slightly under-predicts cancellation and although it is not able to predict the cancellation on the first time period (day 1) as well as MNL, it is capable of predicting cancellation on the last time period (day 16) reasonably well. In term of exchange, DDCM slightly under-predicts the total number of exchanges except for the first (day 1) and the last time period (day 16) which are characterized by a relatively high exchange rate; however, the MNL drastically over predicts exchange decisions throughout all time periods. The prediction of keep obtained from DDCM is reasonably close to the observed value while the MNL significantly under predicts the keep decision as a consequence of over prediction in exchange.

The choice probability for each alternative observed and predicted together with measure of errors for the real data experiment is reported in Table 3. It shows that the D value of the dynamic model is significantly smaller than the correspondent value obtained with the MNL model (1.194 compared to 15.179) indicating a much better prediction capability of the dynamic model over the MNL model.

6. Conclusions and future research directions

This paper has proposed a dynamic discrete choice model for ticket cancellation and exchange with an application in the context of railway ticket purchase for intercity trips. The methodological framework proposed considers forward looking agents that maximize their inter-temporal payoffs when deciding about exchanging or cancelling their ticket. The classical formulation based on the optimal stopping problem derived from dynamic programming is preserved here, while an innovative and elegant scenario tree formulation is proposed to solve the issue of calculating the passengers' expected utility over time. The model is estimated using maximum likelihood estimation, which seems particularly appropriated in this finite horizon problem. The analysis makes an important contribution in the context of discrete choice models for revenue management as it allows accounting for temporal effects on individual decisions that are usually treated in a static context. The model has been successfully on real data from the railway industry; results shows that DDCM outperforms MNL in reproducing the actual decisions.

Several extensions of this work warrant attention. It would be desirable to incorporate the unobservable (or latent class) segments within the population using a discrete segmentation approach; latent class (LC) models in which classes are based on trip characteristics (i.e. as group size, departure time) appears to be well suited for this kind of analysis. The model can also account for taste heterogeneity by incorporating mixed logit choice model (ML) thus allowing for different passenger choice preferences in a

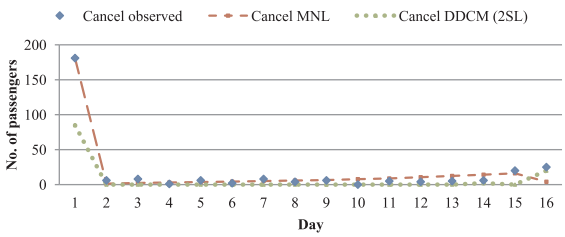

Fig. 3. Validation of cancel decision: real data.

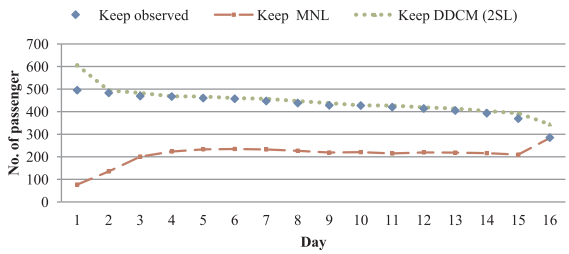

Fig. 4. Validation of keep decision: real data.

Table 3
Model validation: choice probability of real data experiment.

Alternative	Observed	Predicted (Static)	Predicted (Dynamic)
Exchange day 1	0.0287	0.6297	0.0072
Exchange day 2	0.0121	0.7244	0.0020
Exchange day 3	0.0124	0.5797	0.0000
Exchange day 4	0.0021	0.5171	0.0021
Exchange day 5	0.0021	0.4929	0.0000
Exchange day 6	0.0022	0.4804	0.0000
Exchange day 7	0.0044	0.4799	0.0000
Exchange day 8	0.0112	0.4799	0.0000
Exchange day 9	0.0091	0.4856	0.0000
Exchange day 10	0.0024	0.4662	0.0001
Exchange day 11	0.0047	0.4763	0.0000
Exchange day 12	0.0048	0.4521	0.0024
Exchange day 13	0.0097	0.4420	0.0000
Exchange day 14	0.0148	0.4314	0.0000
Exchange day 15	0.0102	0.4254	0.0000
Exchange day 16	0.1599	0.2220	0.0136
Cancel day 1	0.2601	0.2601	0.1221
Cancel day 2	0.0121	0.0026	0.0000
Cancel day 3	0.0166	0.0048	0.0000
Cancel day 4	0.0021	0.0064	0.0000
Cancel day 5	0.0128	0.0079	0.0000
Cancel day 6	0.0043	0.0093	0.0000
Cancel day 7	0.0175	0.0112	0.0000
Cancel day 8	0.0089	0.0130	0.0000
Cancel day 9	0.0137	0.0148	0.0000
Cancel day 10	0.0000	0.0182	0.0000
Cancel day 11	0.0117	0.0206	0.0000
Cancel day 12	0.0095	0.0252	0.0000
Cancel day 13	0.0121	0.0300	0.0000
Cancel day 14	0.0148	0.0351	0.0049
Cancel day 15	0.0509	0.0410	0.0000
Cancel day 16	0.0677	0.0111	0.0542
Keep day 1	0.7112	0.1102	0.8707
Keep day 2	0.9758	0.2729	0.9980
Keep day 3	0.9710	0.4155	1.0000
Keep day 4	0.9957	0.4765	0.9979
Keep day 5	0.9850	0.4991	1.0000
Keep day 6	0.9935	0.5102	1.0000
Keep day 7	0.9781	0.5090	1.0000
Keep day 8	0.9799	0.5072	1.0000
Keep day 9	0.9772	0.4995	1.0000
Keep day 10	0.9976	0.5156	0.9999
Keep day 11	0.9836	0.5030	1.0000
Keep day 12	0.9857	0.5226	0.9976
Keep day 13	0.9783	0.5280	1.0000
Keep day 14	0.9704	0.5336	0.9951
Keep day 15	0.9389	0.5336	1.0000
Keep day 16	0.7723	0.7669	0.9322
D		**15.1790**	**1.1940**

continuous segmentation framework. The current model specification does not account for fare correlation among adjacent departure times and assumes that fares are not dependent on the demand. More complex GEV structures and supply-demand equilibrium models should be considered for a better representation of the exchange cancellation RM problem. Comparison with time proportional odds model used by some researchers for similar case studies would be valuable to test the performances of the two approaches. Finally, the modeling approach applied here to railway revenue management could be applied to test other refund and exchange policies and in general to other problems for which it is relevant to model passenger decision over time.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.tre.2017.12.004.

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